Allometry in Trees

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Abstract

Trees grow in diverse shapes and sizes. It would be interesting to find out why thy grow as they do. So in this project I try to explain the height, branching and general shape of trees. For the height and the branch length an allometric formula is given. Many aspects of the general shapes can be explained by the amazing capability of trees to grow more wood where the tree is stressed.

1 Introduction

Walking nearly anywhere on the surface of the earth, you can see trees in diverse shapes and sizes. But did you ever think about this? Why do they grow as they do? Why did a certain tree (for example that one in front of your window) grow like it did and does look like it does? Of course it is partly because it is a lime tree, which does look quite alike most limetrees. But why are some thin and tall and others thick and short? Why do some grow in funny ways, like the pregnant looking one in Fig.1 left? And why did the tree in Fig.1 right grow around the stone instead of avoiding it altogether?

Whatever makes them grow in these ways, it seems to be a successful concept, because the biggest and most durable organisms on earth are not dinosaurs or wales, but trees. To try to understand the diverse shapes and sizes of trees, we'll start with trying to find an explanation for the proportions of trees.

2 Proportions of trees

The height of big trees follows a cartain basic pattern on which we'll have a closer look now. The diameter of the trunk in relation to the height is bigger in larger trees. In Fig.2 one can easily detect that the diameter of the trunk in relation to the height is bigger in the douglas fir (center) compared to the pine (right) and in the redwood (left) again bigger compared to the douglas fir. This effect can be explained with the help of dimensional analysis.

Dimensional analysis is a conceptual tool used to check the plausibility of derived equations and computations by finding out if the dimensions on both sides of a equation conform. It is also used in physics, chemistry and engineering to understand physical situations involving a mix of different kinds of physical quantities and to form reasonable hypotheses about complex physical situations.

An example is the Mach-Number: The air stream around a plane changes dramatically when it reaches and exceeds the acoustic velocity. At this point

Figure 1: Trees grow in diverse shapes. [www.baumwunder.de]

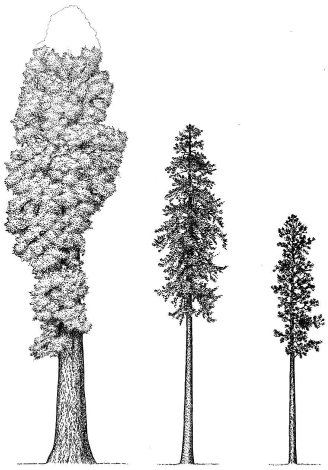

Figure 2: Redwood (left), douglas fir (middle) and pine (right). The diameter of the trunk increases with height.[1]

the plane emits energy, which is hearable as load noise and costs energy which the engine has to muster additionally. The dimesionless relation of flight velocity and acoustic velocity is given by the Mach-number. The acoustic velocity changes with flying altitude and atmospheric pressure, so the need of extra energy does not only depend on the absolute flight velocity, but also very much on the acoustic velocity of the region of the air, the plane is flying in. So it is not a number with a dimension, like the velocity of the plane, which is important to calculate the need of more energy, but a dimensionless number, the Mach-Number.

In the case of trees the important variables for the dimensional analysis are diameter, height, elastic modulus and relative density. This is due to the fact that the elastic modulus is more important for the load capacity than the strength. In winter many trees bend under the load of the snow, but normally regenerate completely in spring. With less elasticity they would break under this load and thereby not be able to regenerate.

Dimensional analysis applied to the bending yields the dimensionless number: $\frac{elastic-modulus \cdot diameter^2}{gravity \cdot relative-density \cdot height^3}$.

As the relation of the material properties elastic modulus and relative density is the same in most living wood, $\frac{diameter^2}{height^3}$ is nearly constant. In allometric terms this is written as $Height \propto Diameter^{\frac{2}{3}}$.

In 1881 Greenhill came to the same conclusion, but with different arguments. He wanted to know how high the cylindric flag pole in Kew Gardens could become before it collapsed. The pole was already 67 metres high and had a diameter of 53 centimetres. With help of the laws of solid mechnics he calculated a height of 91 metres. This finding complies with the conclusion of the dimensional analysis.

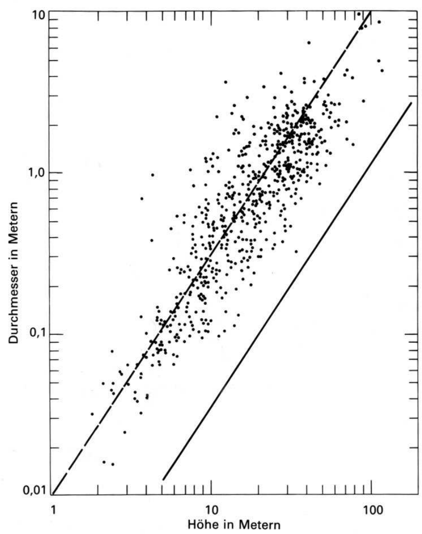

Figure 3: Diameter of trunk plotted against the doubly logarithmic height. These 576 trees represent the highest specimen of one species, wich are known in the USA.[1]

For Fig.2 the diamter of 576 trees was measured 1.52 metres above the ground and plotted against the doubly logarithmic height. The straight line on the right side of the plot describes the height where a cylindric pole with this diameter would collapse. The line of best fit which runs parallel to this line indicates that trees follow the rules of elastic relation.

During growth a tree changes its proportion also complient to the rule of elastic relation. Richard Kiltie and Henry S. Horn measured trees with different heights of the same species (american beech tree). Their measuring points lay also approximately parallel to the straight line.

3 Branches

But trees are more than just a stem. The branches and roots are also statically important factors. As a very simplified model for the branches a rubber stick, which is fixed on one end and bends under its own weight, can be analysed. If the same stick is fixed at different lengths and the pictures are put on top of each other, it is easy to see how sticks with the same diameter but different lengths deflect (Fig.3). The sticks will jut more over the fixed end, the longer they are. But only up to a certain point, where the horizontal distance between the fixed and the bended end reaches a maximum value. This critical length plays a similar role for the branches as the height of an elastic column for the trunk, because functionally it determines an upper bound for the branch length. Just as the height of a trunk has to be smaller as the critical height where it would collaps, the length of a branch is expected to be smaller than the critical length, where the horizontal distance from the tip of the branch to the trunk decreases with more length.

With the help of dimensional analysis it can be shown, that the relation of squared diameter and cubic length is he same for elastic sticks, if the length corresponds to the same fraction of the critical length. Even though the branches of most trees point upwards, this still applies.

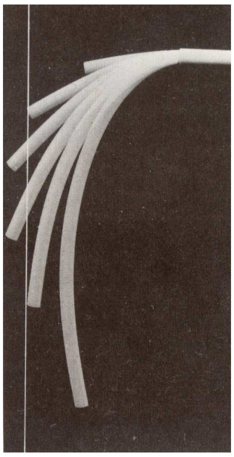

Figure 4: A rubber stick fixed on one end bends on the other end. The bending increases with the length.[1]

To test whether trees are elastically related, one has to gather quite costly measurements. Namely the measuring of the segments between the branching points of a tree and then comparing it with the help of a computer. The local diameters were compared to the length of the whole branches from stem to top and were found related. Again $Height \propto Diameter^{\frac{2}{3}}$, just like in the elastic relation of diameter and overall height.

4 Trees are perfect self-optimizers

Trees have the remarkable capability of reacting to outer stimuli like gravity or wind by thickening according to the stress. Apparently the proprtions of trees are not determined in all details. It seems that the trees rather thicken parts of the stem and branches according to outer stress. This is controlled by the growth hormone auxin, which supports the growth of the cambium. The cambium is a cell layer lying under the bark in plants and trees.

But it is not known, how this is controlled by the trees. Very likely the core rays, which lead radially from the cambium to the central mark play an important role. With this position they are suitable to notice the forces of gravity and wind resulting in the ramming on one side of the stem or branch and the stretching on the other side. So the core rays might act like sensors for these forces and might communicate to the cambium where to start secondary growth. Up to now there are neither arguments for nor against this hypotheses. However, from observations it is known, that trees, growing in the greenhouse increase more in perimeter if they are bend five to ten times a day. And on the other hand, trees ouside should not be supported for too long, because they will grow very fast and might not be able to stand on their own afterwards. In some cases, one can observe, that the stem grows even thicker above than below the fixation.

Regardless of which mechanisms make the tree thicken, the diameter of stem and branches is determined at least partially by the flexure stress. Because of this remarkable capability, the stems represent a mechanical optimum with respect to tapering, branch and root junctions, and inner architecture. In other words, one could say that trees are perfect self-adjusting optimizers, because they grow according to the forces and aim at an even distribution of the mechanical stresses.

With this in mind, a lot of the features of trees can be explained. An example is that in most trees the diameter of the trunk increases downwards, which can be explained by the very high shear force in the transition of trunk and radix, where tractive efforts and compressive forces cross (Fig.4).

In bifurcations the same forces take effect (Fig.4).

Only vital trees with a low top have an increasing diameter. Closely standing trees in the woods grow very tall and at the same time thin to get sun and are likely to collaps (Fig.4). As a rule of thump, trees with a relation $\frac{height}{diameter} > 50$ are likely to collaps. In Fig.4 the relation of height and diameter is plotted against their frequency with still standing trees in green and collapsed trees in red. So when a clearing develops in the woods, the forrest warden will try to grow back trees as soon as possible, because otherwise more trees - standing alone now - would collaps and the clearing would grow.

The roots also grow according to the forces: The roots have to stand bending

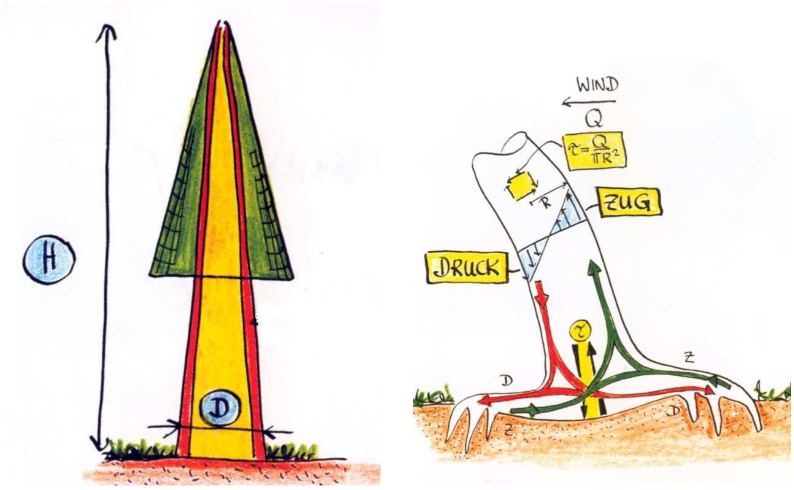

Figure 5: The diameter of the stem increases with size. This can be explained by the high shear forces in the transition of trunk and radix.[2]

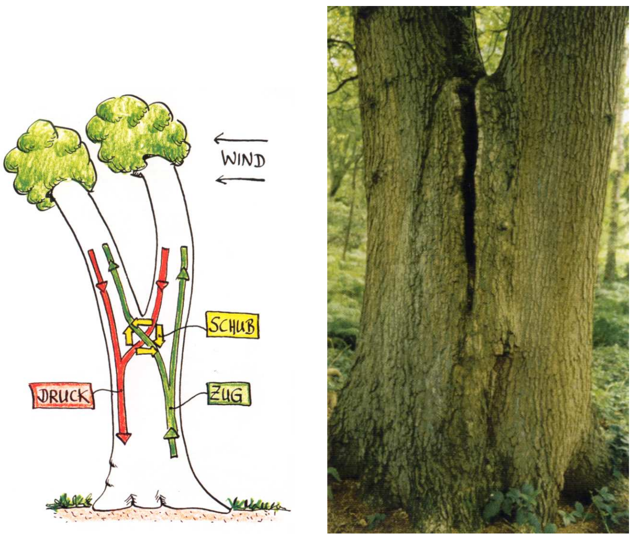

Figure 6: In bifurcations high shear forces act. This might induce breakage.[2]

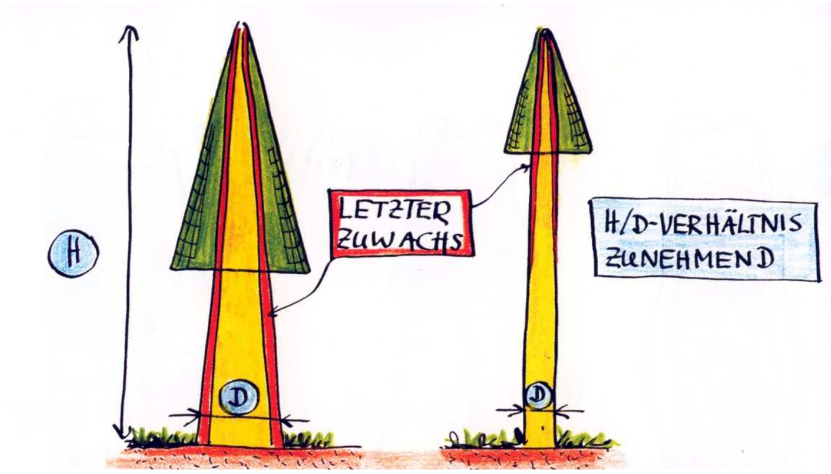

Figure 7: Only vital trees with a low top have an increasing diameter.[2]

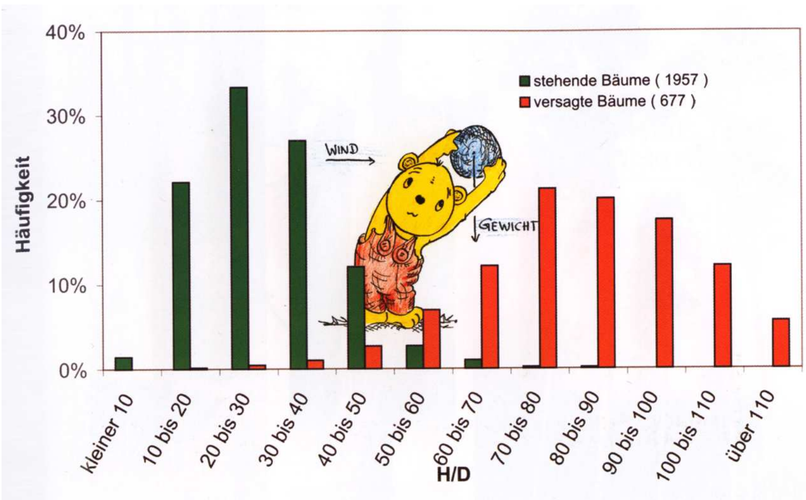

Figure 8: As a rule of thump, trees with the relation height/diameter ¿ 50 are likely to collaps.[3]

stresses which do not change the direction of the stress, but only act in one direction. So many roots form an eight, which is a nearly optimal profile for this. In Fig.4 there is a root depicted on the right next to a structural element, called I-Beam. This structural element is also nearly optimal for bending stresses, because alike the root there is less material in the middle, where less forces act, but more at the edges were the bending forces act.

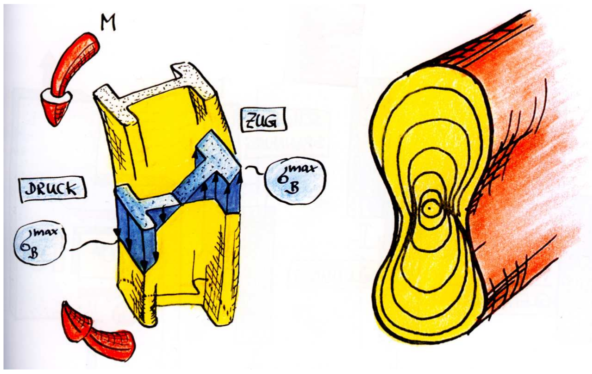

Figure 9: An I-Beam (left) is nearly optimal for bending stresses coming from one direction. So is an eight-formed radix (right).[2]

With the striving for an even distribution of the mechanical stresses, the examples of the beginning can also be explained: When a stone presses against a tree, the high contact forces will be evenly distributed if the tree grows around it (Fig.4 left). The pregnant tree was most likely hollow, so that it grew more wood around the hole, to reduce the weakness of this plaw (Fig.4 right).

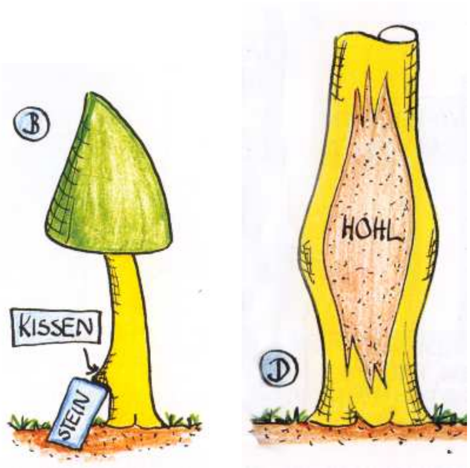

Figure 10: Tree growing around a stone (left), hollow tree (right)[2]

References

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