Population Dynamics: A Catastrophe Model For Fishing

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Abstract

This article decribes how models of population dynamics can help to make the right decisions in fishery management. The main topic is the development of a model for the population dynamics of an anchovies population in the southern pacific off shore of Peru. At first I will give an introduction to the problem that will be the basis for our model and I will decribe the model step by step. The second part consists of a detailed analysis of the model's properties which will show the strength but also the limitations of the use of population-dynamics in fishery management.

1 The Use of Population Dynamics in Real World Problems

Fishery management nowadays is one of the most important fields that benefits from mathematical modeling of population dynamics. The prediction of the development of fish populations under different fishing intensities is a crucial step in specifying fishing quotas for certain species.

The example I am going to present shows the severe consequences if the prediction is wrong or no prediction is used at all.

In the middle of the 20th century there was a large boom in the peruvian fishery market. Until then there were mostly independent fishers with small boats and peru did not export anchovies at all. But from the beginning of the 1950s the export grew stronger and stronger when big companies moved into the market and pushed the fishing effort and with it the harvest to never experienced heights. Nearly every year the export of anchovies showed strong growth rates and the whole peruvian economy became more and more dependent on the foreign currency from the fishing exports. This process continued until 1973, when there was a sudden breakdown of the numbers of harvested fish. In one year the exported tons of anchovies dropped from about 12 million to less than 2 million tons. The population collapsed from one year to the other. This lead to the collapse of the fishing industry at the coast of Peru and with it the economy of the whole county suffered a severe crisis. In the years following the breakdown the harvested tons of fish did not recover from the crash and so most companies left the unprofitable fishing grounds. Although fishing was reduced to a minimum in those years, the population did not recover until the 1990s (see figure 1)[1]

To get an explanation for the sudden breakdown of the population and its slow recovery, we use the approach of modeling the system with methods of

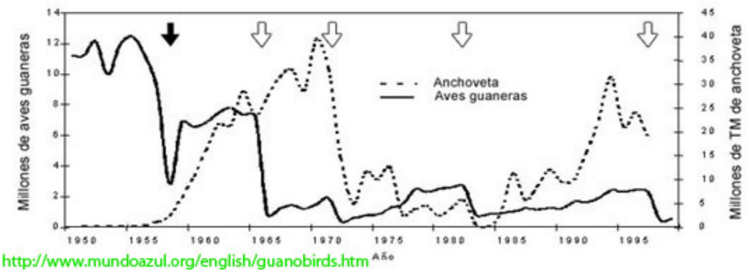

Figure 1: The dotted line shows the exported tons(in millions) of anchovies per year. The breakdown in 1973 can be seen very clearly. The other graph shows the consequences for the whole ecosystem. It is the production of guano, which is produced by seabirds. These birds are predators of the anchovies, and their population had a breakdown even before the anchovies population was depleted.

population dynamics. For a model of this system we have to take into account two main factors: Biology and Economy. The most important biological factor is the growth rate which depends on the population size and the amount of harvested fish. From the economic point of view we have to look at the fishing effort which depends on the costs per effort and the price for fish.

Let us first examine the biological part of the model, which is most important to us. Because anchovies have overlapping populations, we can use a continuos population model. One of the simplest formulas for a realistic computation of the growth rate is the logistic equation.

$$x' = rx(1 - \frac{x}{K}) \tag{1}$$

Where r is a linear growth factor and K is the maximal population size without harvesting. This equation leads to a sigmoidal development of the population. We could use this equation to describe the growth rate of the anchovies, but we will get more realistic results if we take into account another important factor for the development of the population. Anchovies live in huge swarms, and simplified we can say that the bigger the swarm is, the higher is the fitness of its individuals which leads to higher growth rates in bigger swarms. We can include this factor into our formula by changing it in the following way:

$$x' = rx^2(1 - \frac{x}{K}) \tag{2}$$

Which leads to stronger sigmoidality of the curve as can be seen in figure 1.

Now we have a differential equation for the change of the population-size without any fishing efforts. We can now include fishing into the model by subtracting the harvested amount of fish. We also add a constant factor a to model a immigration of fishes from other parts of the ocean, which prevents the population from complete distinction. This yields the equation:

$$x' = a + rx^2(1 - \frac{x}{K}) - vEx \tag{3}$$

Where v is the the gain of fish per effort unit E.

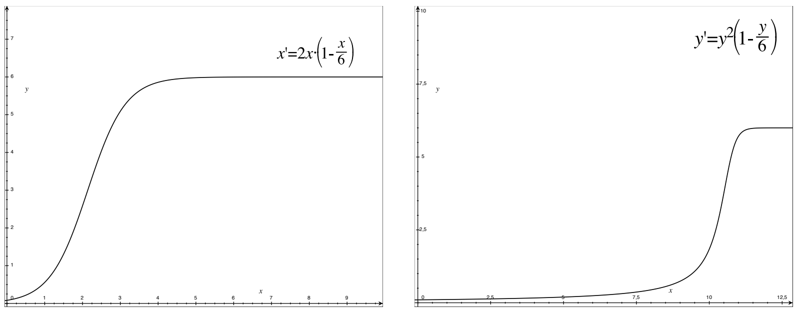

Figure 2: The graphs show the difference between the formulas 1 and 2.

This leads us to the very simplified model of the economy which will be sufficient for our needs. The fishing effort E depends on simple market rules. If we can get large harvests per effort unit, the effort will grow, otherwise it will decrease. The turning-point depends on the parameter p which is the price on the market, c which is the cost per effort unit, and v. This can be modeled by the following equation:

$$E' = \alpha E(pvx - c) \tag{4}$$

E is a slow changing variable, because the the fishery industry needs some time to increase or decrease fishing efforts. This is represented by the small factor α. Now we have a system of two differential equations that roughly models the problem described above. In the following we are going to analyze the model and we will try to explain the behavior of the anchovies population.

2 Simulation and Analysis of the model

At first we will simulate the model, which will show if the equations at least roughly resemble reality. The time course simulation of the model clearly shows that this is the case. (see fig. 2)‘

2.1 Steady States and Bifurcationanalyses

Now we will analyze the properties of the model more deeply. At first we take a look at the steady states. For this purpose we assume that the fishing effort is a fixed variable and find the steady states graphically for different values for E. The intersections of both curves are steady states of the system, because we subtract E from the growth rate without fishing, and if both are equal there can be no change. The analysis shows that we have a different number of steady states at different values of fishing effort. (see fig. 2.1)

This already explains the rapid breakdown of the population at a certain level of fishing effort, because there is a big difference between the upper stable state an the lower stable state. There is a value for E where this jump happens, but to determine this value in real world problems is very hard.

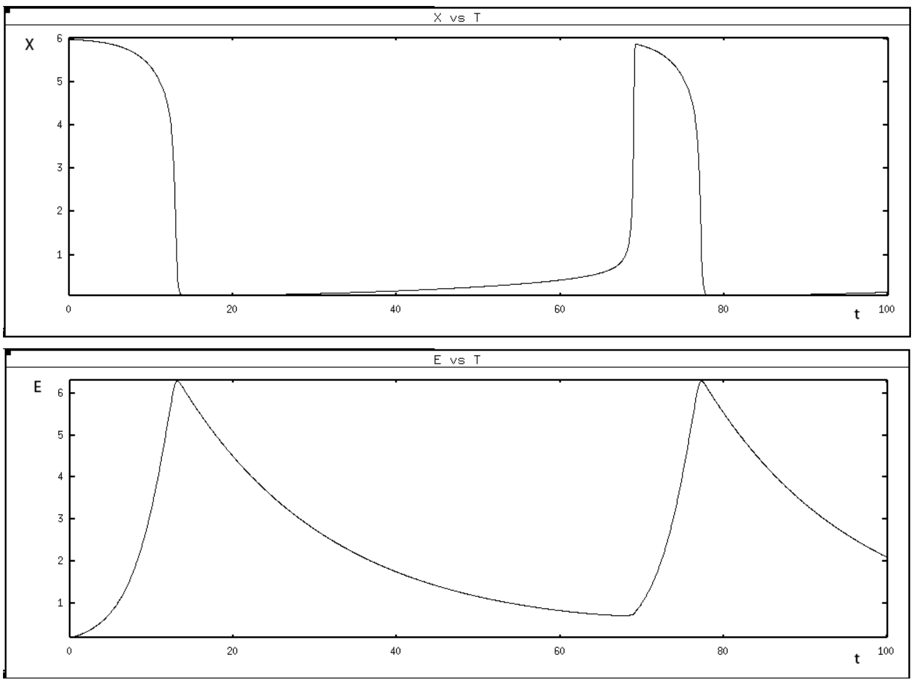

Figure 3: The upper graph shows the simulation of the population size over time while the lower curve shows the development of fishing efforts. We can clearly observe the sudden breakdown of population size which is most important property that the model shall exhibit.

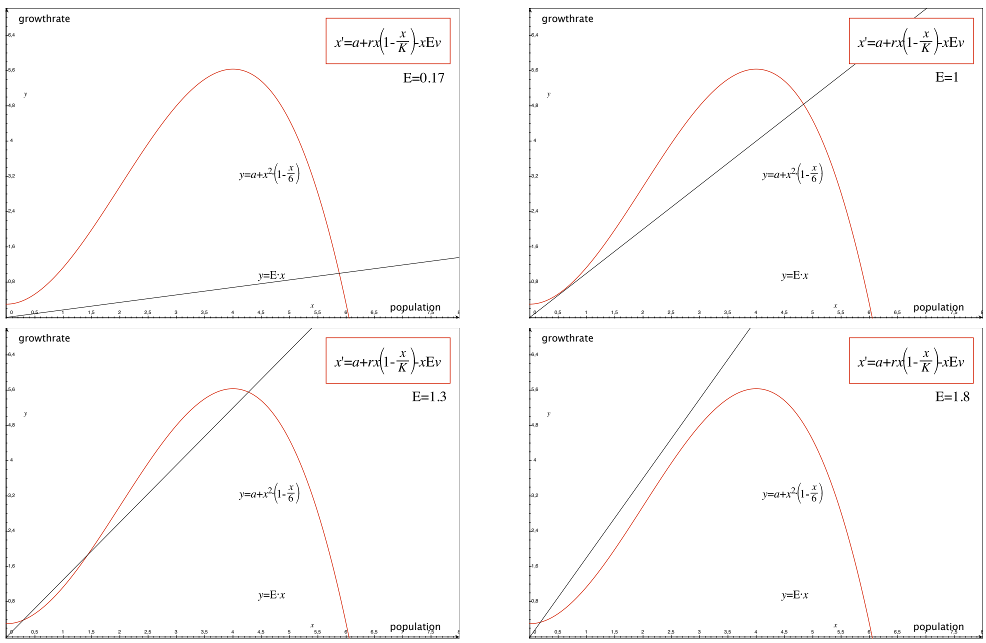

Figure 4: These graphs show the different steady states of the system. At $E = 0.17$ there is only one steady state at a high population level. (upper left) At a value of $E = 1$ there is a bifurcation and we get a new steady state at a much lower population level. (upper right) Between $E = 1.3$ and $E = 1.6$ there are three steady states, of which two are stable and the one in the middle is unstable. For all values over 1.6 there is only one stable steady state near the minimal population level a. (lower left, lower right)

2.2 Phaseplane and Hysteresis

We will now look at the phaseplane of the system with both variables depending on each other. The phaseplane makes the development of the system very clear. The model starts near the maximum population and nearly zero fishing efforts. At this point the fishing industry makes high profits, because the population x is very high. This causes the effort E to increase, but because of the factor α we introduced it cannot increase rapidly. Meanwhile the population of fishes decreases slowly. This slow decrease corresponds to the steady state at a high population as seen in figure 2.1. Then there is a sudden breakdown in populationsize to nearly zero, which causes the efforts to decrease. This breakdown corresponds to the bifurcation point in which the three steady states join to one stable steady state at a low value of x (see figure 2.1). This event is often called catastrophe, because there is no easy way back to higher population levels. [2] The system exhibits hysteresis, which is a very important property that can also be found in the real population. Hysteresis means that the immediate history of the system determines its current state and can be thought of as a memory of the system. When the fishing efforts are reduced to the level before the breakdown, the population does not go back to the higher levels, but stays at the very low level. The breakdown is nearly irreversible, and unless fishing efforts are reduced to nearly zero the population does not recover. But the strong hysteresis effect is not the worst thing that can happen to a system like this.

In our model, only the constant a makes a return to higher population levels possible. This constant prevents the total extinction of the species, and as the fishing efforts decrease the population can recover. The recovery starts very slowly and grows exponentially as E approaches a critical value. But there is not necessarily such a constant. We introduced the factor a a migration of fishes from other parts of the ocean. But if we think of a lake instead, there is no such migration and the species goes extinct. The difference can be seen in figure 2.2.

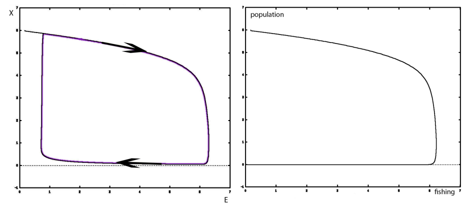

Figure 5: Both graphs show the phaseplane of the model, but while the left model uses the constant a to prevent extinction, the right graph does not. So the system reaches a steady state a the origin after the catastrophe.

3 Maximum Sustainable Yield

With this model it is now possible to predict a level of fishing effort that is most profitable for the fishery industry and at the same time prevents the system from catastrophe. But this is a very complex task with many difficulties. The first problem is, that we can never get exact data about the current state of our system, which is true for the population as well as fishing efforts. For the population, it is simply impossible to measure it directly, so it has to be extrapolated from the harvest and some punctual measurements. The fishing effort is easier to measure, but there can still be other variables that suddenly alter the yield, for example there could be a more effective fishing technique which suddenly increases the gain per yield v and therefore increases the harvest without bigger fishing efforts.

Even if we are able to assemble reliable data and can find a good parameter fit to it, it is still hard to determine the maximum sustainable yield (MSY) for a system [3]. That is because we can have big uncertainties like climate changes, which become more important for our system as it reaches the MSY. The system becomes more and more unstable with higher fishing efforts as it is shown in figure 3.

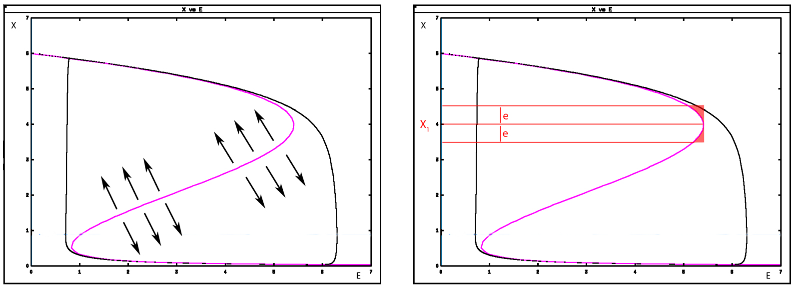

Figure 6: The left figure shows the separatrix of the system in purple. Above this line the population recovers to the high level steady state, below the separatrix it drops down to the low equilibrium. The right image shows the that the system is robust at low levels of fishing, and can sustain a high population even if we introduce some uncertainty e. This changes if the fishing effort is increased and we get below the separatrix even with very small fluctuations.

There are many different methods for predicting the MSY, but they should all be used very carefully and with big confidence intervals to prevent breakdowns. The difficulty for governments which have to determine fishing quotas is to get a good compromise between short time profits and long lasting fishing grounds. Mathematical modeling is an indispensable tool to find such a compromise.

References

[1] Edward Beltrami. *Von Krebsen und Kriminellen. Mathematische Modelle in Biologie und Soziologie.* Vieweg, 1993.

[2] Wright. Catastrophe theory in management forecasting and decision making. *J. Opl Res. Soc.*, 1983.

[3] Peterman. A simple mechanism that causes collabsing stability regions in exploited salmoid populations. *J. Fish. Res. Bd. Can.*, 1977.