Introduction on Development of Pattern

1. Instability - breakdown of symmetry and homogeneity

2. Stability of a stationary State

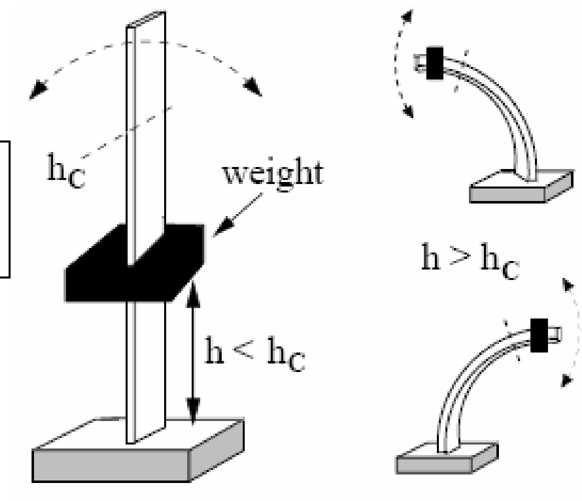
3. Diffusion enhances/suppresses induced local instability

4. Examination of Pattern Development

5. Summary & Outlook

1.Breakdown of Symmetry and Homogeneity

A quite homogeneous system (prepattern) develops a pattern (spatial differences) due to an instability of former homogenous equilibrium / stationary state



[1]

Prepattern –> Small Perturbation + driving Force (Gravity) -> new Pattern(stable)

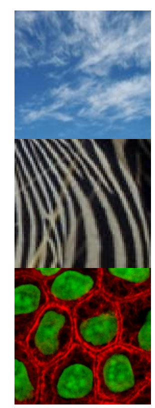
On various levels:

-gastrulation (early phase in the development of animal embryos)

-shape clouds (climate zone)

-fur of a zebra

-phyllotaxis (sunlight, wind, etc)



1.Breakdown of Symmetry and Homogeneity

Pattern formation as a result of the breakdown of symmetry and homogeneity within a system

Turing investigates the instability using a reaction-diffusion system [2].

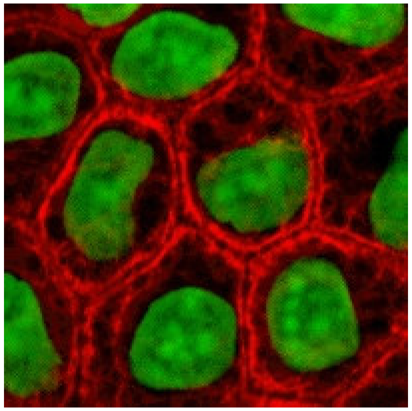
This System consists of:

-chemical substances (2 morphogens).

-simple rules of chemical reactions

-cell walls (limits the system locally)

-laws of diffusion (new stable pattern of concentration)



Cells in culture, stained for keratin (red) and DNA (green).[3]

2. Stability of a stationary State

Stability of a stationary State (Diffusion is neglected)

Reaction Kinetics

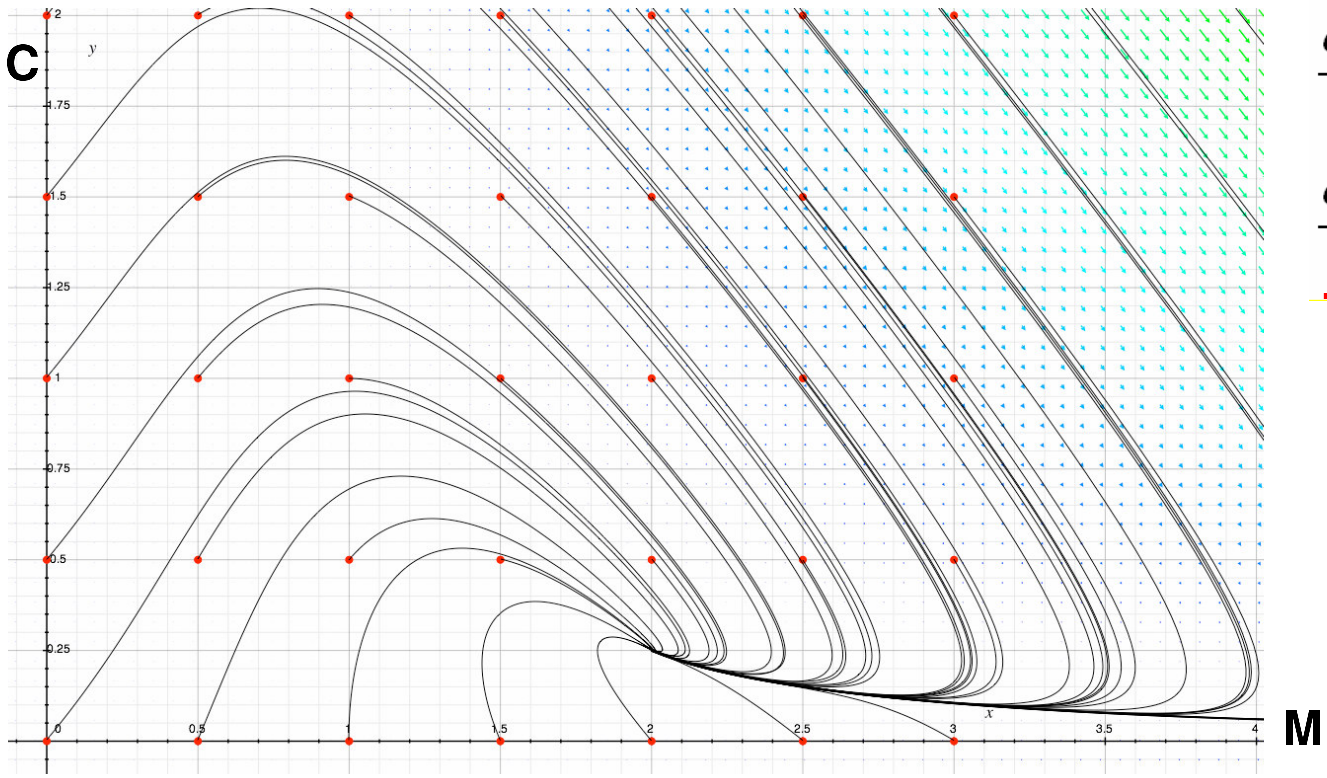
Cannibals C and missionaries M
-M recruited from the outer world as celibates until
Death (by **a**)
-C also die but can reproduce (by **b**)
-2 M meet C, C becomes M

When mixed together M and C reach **equilibrium**
C can be seen as an catalyst, and M as a “poison” [Turing]

$$\frac{dM}{dt} = a - M + M^2 C$$
$$\frac{dC}{dt} = b - M^2 C$$

[3]

2. Stability of a stationary State



$$\frac{dM}{dt} = a - M + M^2 C$$

$$\frac{dC}{dt} = b - M^2 C$$

M=1:3 C=1:3 a=b=1 stable steady state =(2,0.25) (equilibrium)

Eigenvalues:

-0.499750 + i 0.866170

-0.499750 + i -0.866170

2. Stability of a stationary State

(Diffusion is neglected)

$$\frac{dM}{dt} = a - M + M^2 C$$

$$\frac{dC}{dt} = b - M^2 C$$

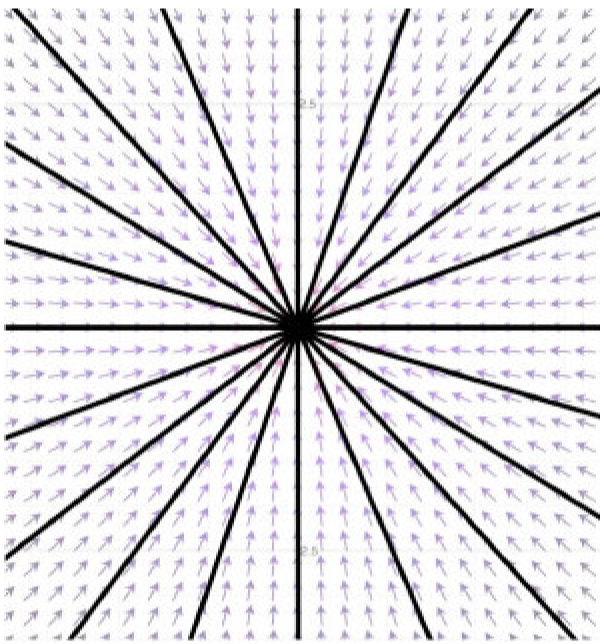
Reaction (Real Eigenvalues)

stable critical point tends system to stability

$\lambda_1\ \lambda_2 > 0$ (source, unstable)

$\lambda_1\ \lambda_2 < 0$ (sink, stable)

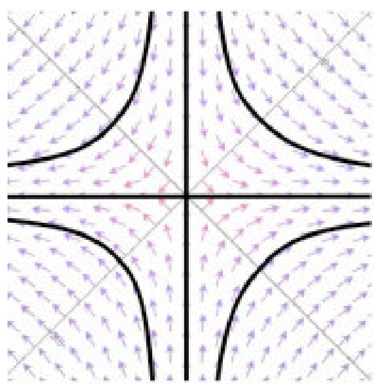
$\lambda_1 = \lambda_2$(focus)



Saddlepoint

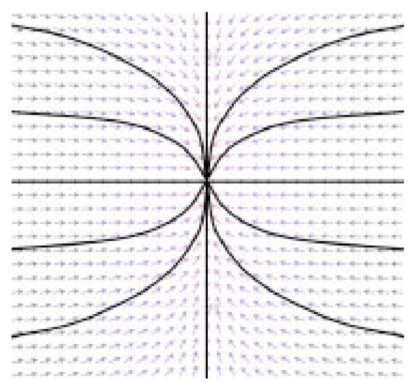
(unstable)

$\lambda 1 / \lambda 2 < 0$



Stable Node

$\lambda 1 / \lambda 2 > 0$



[5]

2. Stability of a stationary State

Stability of a stationary State Diffusion is still neglected

Taylorreihe -> jacobian matrix -> properties eigenvalue on stationarity

Properties of Complex Eigenvalues within a linear differential equation.

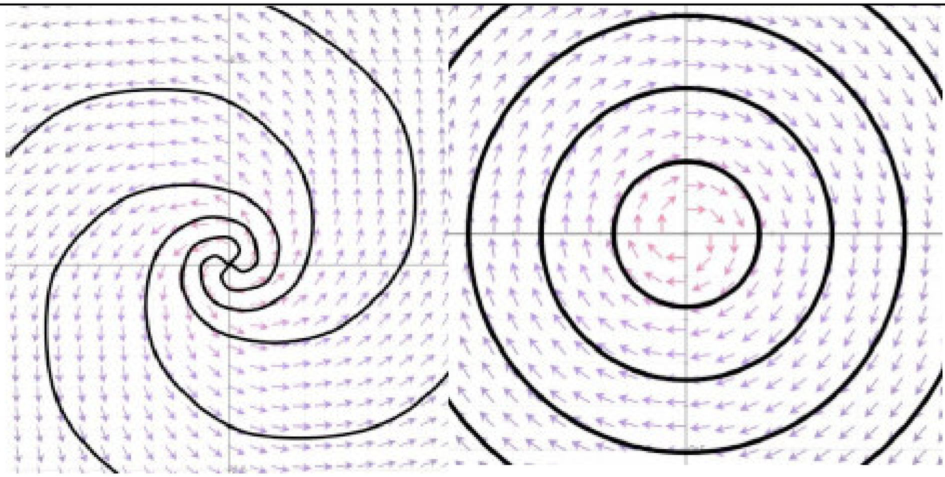
Complex Eigenvalues **λ =** a+i or a-i (a is real part)

a=0 (Center);

a<0 (inward spiral, stable);

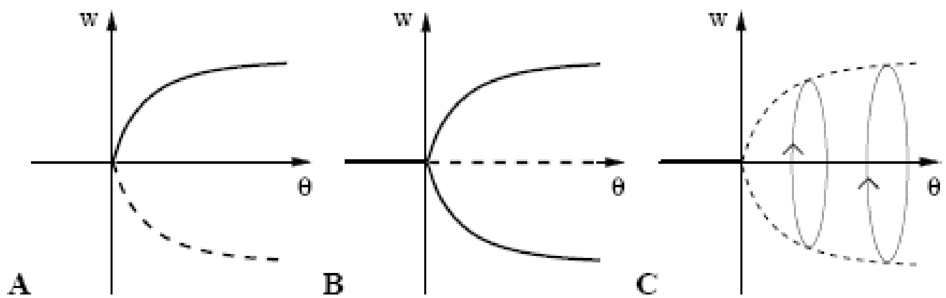
a>0 (outward spiral, unstable)

M=1:5 C=1:5 a=b=1
stable steady state
(2,0.25) (equilibrium)
Eigenvalues:
-0.499750 + i 0.866170
-0.499750 + i -0.866170

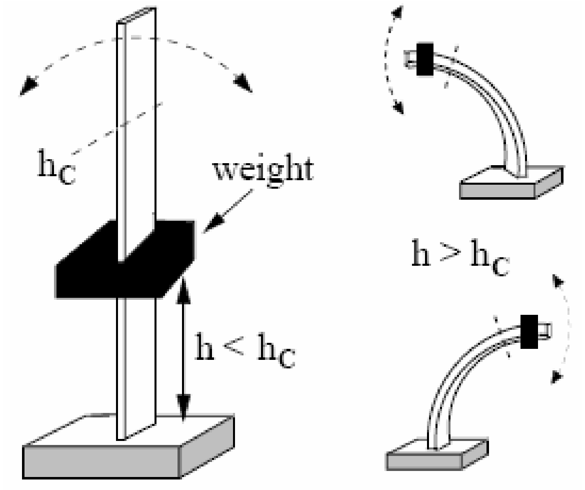


<=[5]

2. Stability of a stationary State



[1]



A Saddle Node Bifurcation, B Pitchfork Bifurcation, C Hopfbifurcation

Turing instability/bifurcation, number of stationary states do not change

3.Diffusion induced, local instability (enhanced)

-C and M are spread out on the beach of an island, diffusing

-M move faster (lets say they have bikes, by d>1)

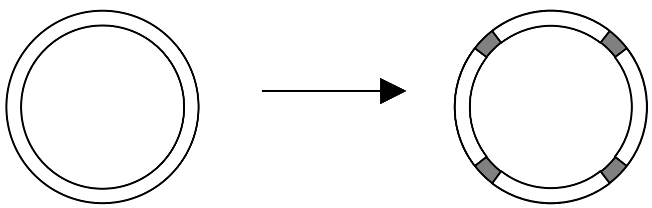
-local excess of C will produce more C and M

-a high low pattern and a length scale appear

$$\frac{\partial M}{\partial t} = a - M + M^2 C + d\nabla^2 M$$

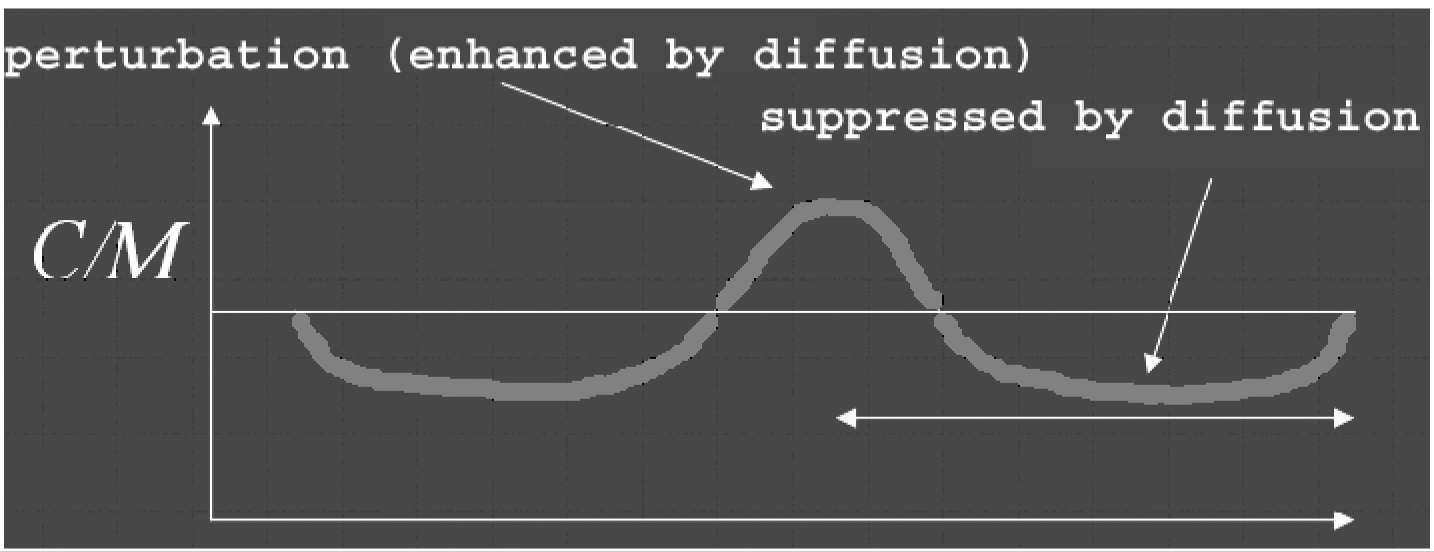
$$\frac{\partial C}{\partial t} = b - M^2 C + \nabla^2 C$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2}$$



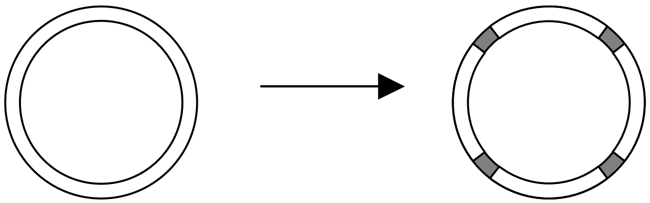
Reaction Kinetics

Diffusionterms



[7]

4.Examination of Pattern Development



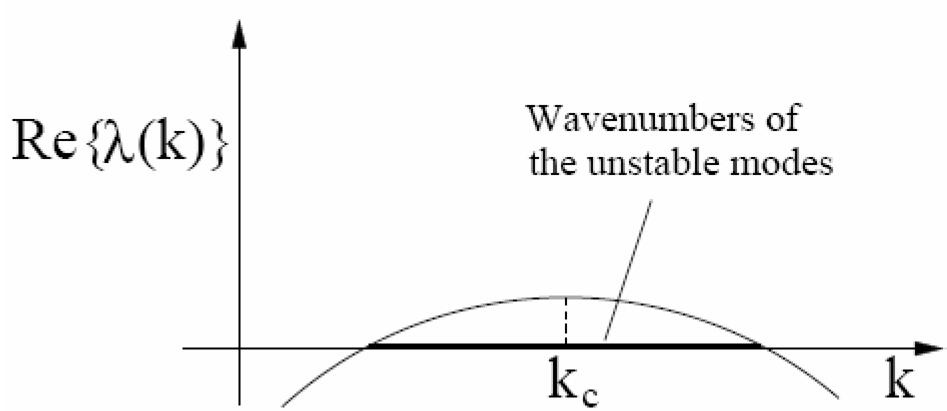
General term:reaction-diffusion system

$$\frac{\partial \mathbf{w}}{\partial t} = \mathbf{D}\,\nabla^2\mathbf{w} + \mathbf{F}(\mathbf{w},\Theta),$$

[1]

Perturbation/concentration Bifurcation parameter

„Dispersionanalysis“



[1]

$$k_c^2 = \frac{D_V f_U + D_U g_V}{2 D_U D_V} = \sqrt{\frac{f_U g_V - f_V g_U}{D_U D_V}},$$

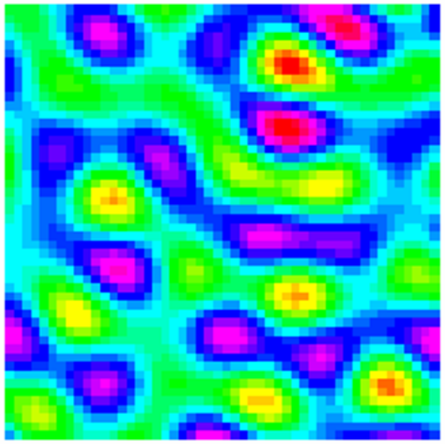
$$\begin{aligned} f_U + g_V &< 0 \\ f_U g_V - f_V g_U &> 0 \\ D_V f_U + D_U g_V &> 0. \end{aligned}$$

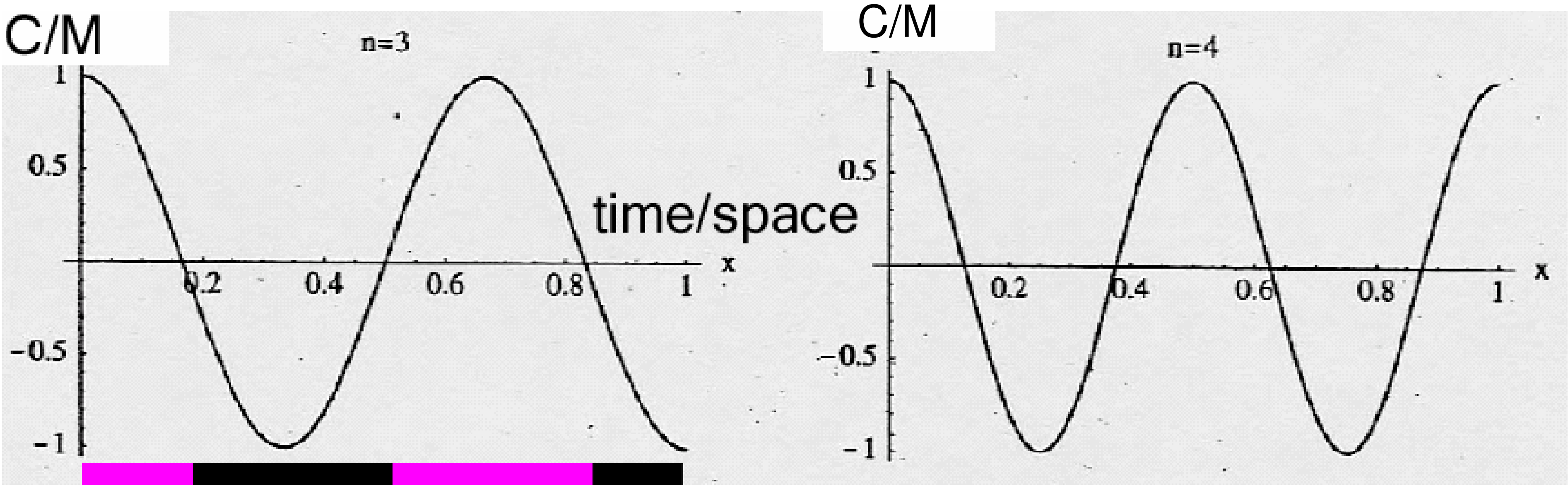
4.Examination of Pattern Development

A periodical pattern of concentration is induced mediated by diffusion on a local instability.

The Wavelength of the pattern, depended on **constants of the reactions** and the **coefficients of the diffusion**.

Not on the geometry of the system





5.Summary & Outlook

-patterns are spatial or temporal differences

-stability of prepattern(reaction),
ability of a system to resist small perturbations

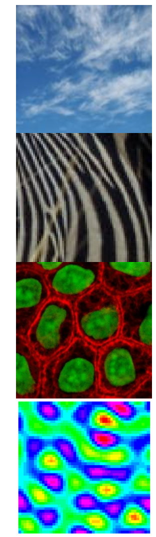
-simple reaction diffusion system

-local instability and global stability (diffusion)

-wavelength/perturbation dependent on parameter of the system

5.Summary & Outlook

Arising Qestions:



-What interactions/processes between 2 Morphogens(Cannibals ,Missionaries on more islands) are possible,

- What interactions/processes else are necessary to develop complex patterns?

-What is the source of local instability ?

Sources

1. „Computational Studies of Pattern Formation in Turing Systems“ , Teemu Leepänen, http://lib.tkk.fi/Diss/2004/isbn9512273969/isbn9512273969.pdf

2. „The chemical basis of mophogenesis“, A.M. Turing; Philiosophical Transactions of the Royal Society of London, Series B, Biological Sciences, Vol 237, No. 641 (1952),pp. 37-72.
3. Picture of a cell http://en.wikipedia.org/wiki/Cell_%28biology%29.
4. http://www.swintons.net/deodands/archives/000091.html
5. http://www.zib.de/visual/events/SciVisLectureSS07/lecture8.pdf
6. „Modellierung biologischer Systeme“, HU 2000
7. http://www.swintons.net/jonathan/Turing/talks/Manchester%202004.ppt
8. Gierer A. Meinhardt H.: A generative principle of pattern formation based on lateral inhibition, local instability and global stability

Outtakes

Diffusion flow is now calculated.

But not from cell to cell but within, there is no diffusion transfer at the border.

Diffusion Flow

jA=-DA*dA/dx (Fickschen Law)

[Eventuell Tafel vorrechnen dauert aber zu lange]
du/dt= fa(Ao,Bo)u + fb(Ao,Bo)v + Da(d\^2u/dx\^2)
dv/dt= fv(Ao,Bo)u + gb(Ao,Bo)v + Db(d\^2v/dx\^2)

now small gradients can occur which lead to diffusion flows

Dispersionsrelation

Most general term for turing raction diffusion system

$$\frac{\partial \mathbf{w}}{\partial t} = \mathbf{D}\,\nabla^2\mathbf{w} + \mathbf{F}(\mathbf{w}, \Theta),$$

W=concetration fields, theta bifurcation parameter, f describing reaction kinetics

Perturbation (within the feedrate of the system)

$$\mathbf{w} = \mathbf{w}_0 + \mathbf{dw} \qquad \mathbf{dw}(x, t) = \sum_j c_j e^{\lambda_j t} e^{-ik_j \cdot x}. \qquad \mathbf{w} = (U, V)^T \qquad \lambda_j = \lambda(k_j)$$

Dispersions relation calculating k

$$\lambda^2 + \left[(D_U + D_V)k^2 - f_U - g_V\right]\lambda + D_U D_V k^4 - k^2(D_V f_U + D_U g_V) + f_U g_V - f_V g_U = 0.$$

Outtakes

Simple system description with two morphogens and 2 cells:

i) a set of reactions producing the first morphogen with rate "I" (second analog)

satisfied by: **A->X, B->Y**

(A & B substances of continually present, fuel substances)

ii) a set destroying the second morphogen (Y) at rate 7Y

satisfied by: **Y->D**

(D being a inert substance , waste product)

iii) a set converting X into Y at rate 6X

satisfied by: **X->Y+E**

iv) a set of producing X at the rate of 11X

A+X->U U->2X

(X a catalyst of its own formation, U a unstable compound)

v) a set destroying X at a rate of 6Y

X+C->V; V+Y->W; W->C+H

(catalyst C decreasing X is being created out of X)