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# Catastrophe Theory in Management Forecasting and Decision Making

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This paper reviews the use of catastrophe theory in management. Two case studies of practical applications of catastrophe theory in the management of natural resource systems are described. These are used as a basis for discussing the possible use of catastrophe theory in other areas of management.

## INTRODUCTION

THE PURPOSE of this paper is to indicate the relevance of catastrophe theory to forecasting and decision making. Two practical applications are presented in the form of case studies. The potential for further use of catastrophe theory in other areas of management is discussed by reference to these case studies.

Catastrophe theory is a term describing the application of Thom's abstract mathematical theory of structural stability[1] to practical systems. The great attraction of this theory in its first applications was that it dealt qualitatively rather than quantitatively with the system. Thus it was applied[2] in such areas as psychology and sociology, where some variables are not easily quantified. It is important in discussing this issue to bear in mind whether one needs a quantitative or qualitative model of one's system.

A qualitative description can be given in catastrophe theory by representing only the topological structure of the system. There are two very elementary topological structures which constitute the vast majority of applications in the literature. The first is the Fold, see Figure 1, which illustrates the equilibrium points. The system is described by a state variable, $x$, and a parameter, $a$. If we start at the stable equilibrium, $A$, and $a$ increases, the system will move from $A$ to $B$ and then a catastrophe will occur. If $a$ increases further, the system will jump to the only available stable equilibrium, $C$, and then continue to $D$. We call this jump a catastrophe. If $a$ is now decreased, the system will not return along the same path. Instead we observe some hysteresis in that the return path is different: $D$, $C$, $E$, $F$, $A$, with another catastrophe at $E$. The significance of the dotted line, $EB$, of unstable equilibria is that it provides a 'breakpoint curve'. If the system is displaced from equilibrium to a point, $X$, above the breakpoint curve, then it will home in on an upper stable equilibrium on $EC$. However, if the system is displaced to a point, $Y$, below the breakpoint curve, it will home in on a lower stable equilibrium on $FB$. In this description it is implicit that the time constant for attaining an equilibrium value of $x$ is short compared to that for the variation of $a$. This difference in time constants is essential for the application of catastrophe theory and is an important point to bear in mind in considering potential applications. The above description of a system with variables $x$ and $a$ can easily be derived quantitatively from, for instance, the minimization of the potential function:

$$V = x^4 + x^3 - ax.$$

The minima correspond to the stable equilibria discussed above. In this sense, catastrophe theory has not added anything to a simple exercise in calculus. There are two senses in which it has contributed. Firstly the description is qualitative and can therefore be used in systems for which we do not have an algebraic potential function or for which some of the variables cannot be quantified. Secondly, the point of Thom's theory is that the topological form of a fold with two catastrophes and hysteresis is the same, independent of the dimension of $x$. Thus we can bundle into $x$ a whole set of variables, many of which

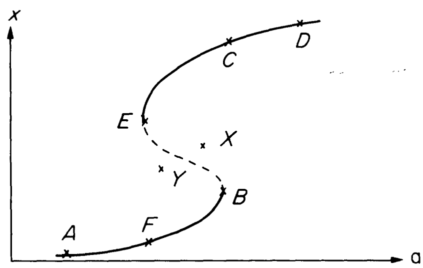


FIG. 1. *The fold.*

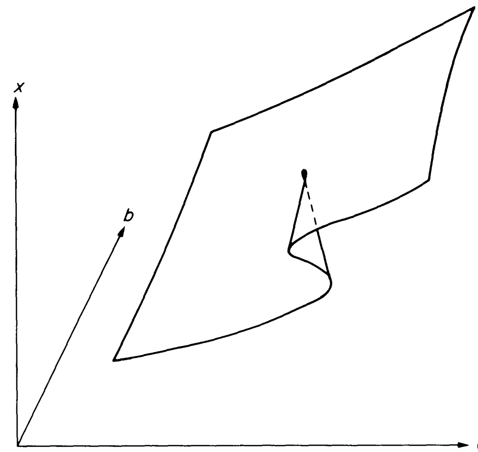


FIG. 2. *The cusp.*

may be difficult to define, let alone measure, and they will all exhibit the behaviour described above when a (scalar) parameter $a$ is varied. In the two case studies presented in the next section, great use has been made of this point to simplify the models. It is a tendency in much modelling that the development of a model is virtually synonymous with increasing the number of variables and complexity of the model. In contrast to this, the models described in the next section have evolved by reducing the number of variables, observing that the qualitative behaviour of many of them is the same. They are in this sense modelled as a single variable.

The second type of topological structure that is important in applications is the Cusp, see Figure 2. This is essentially an extension of the Fold to include another parameter, $b$. When $b$ is below a certain critical value, the system will exhibit the Fold behaviour, described above, but for higher values of $b$ the behaviour of the system is quite smooth (the equilibrium value of $x$ being a continuous function of $a$), exhibiting no catastrophes.

## CATASTROPHE THEORY IN NATURAL RESOURCE MANAGEMENT

The feature of the systems described above which gives rise to their catastrophic behaviour is the presence of two different stable equilibrium values of $x$ for certain given values of $a$, $b$. Ecosystems with predator–prey relationships within them often exhibit two possible stable states, one with a low population level of both predator and prey and the other at a high level. There is therefore the possibility of a catastrophic jump from one level to the other. This is very important in the management of animal or plant populations as natural resources. We now consider one example from forestry management and another from fishery management.

### (a) *Forestry management*

We now present a slightly simplified account of a detailed investigation by Holling[3] of the effect of an insect pest, the spruce budworm, on coniferous forest in Canada. This pest sometimes occurs at epidemic levels in the forest, causing severe defoliation. The purpose of the study is to prescribe a management strategy for controlling the budworm by choosing a combination of the alternatives: (i) tree felling, (ii) spraying with insecticide.

The population, $B$, of the budworm is controlled by two factors: (i) birds, which prey on the budworm, (ii) amount of leaf area, since leaves are the budworm's food.

This is shown qualitatively in Figure 3, the graph dividing into three distinct regions:

*P*. so few budworms, the birds cannot find them;
*Q*. birds eat budworms faster then the latter reproduce;
*R*. budworms reproduce faster than birds can eat them. As the number of budworms increases towards M, their rate of reproduction becomes limited by the food supply.

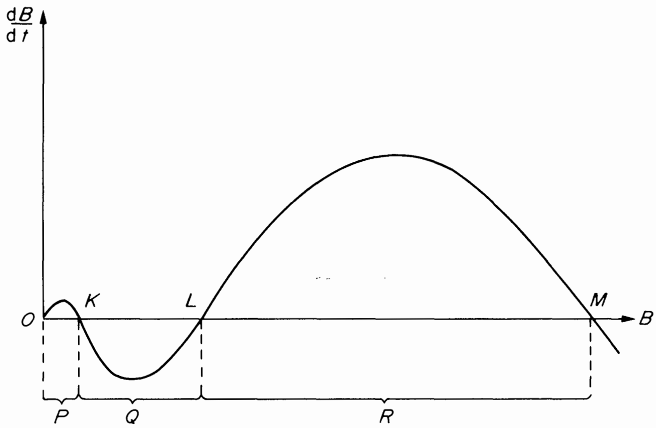


FIG. 3. *Effect of predation by birds on budworm population*, B (*derived from Holling*[3]).

There are four possible equilibrium budworm populations: at $O$, $K$, $L$, $M$; given by $dB/dt = 0$. Of these, the stable budworm populations, constrained by predation and food supply, are at $K$ and $M$ respectively; $O$, $L$ represent unstable populations.

As the trees grow older, the leaf area increases. The effect of this on the budworm population is shown in Figure 4. It will be seen that, when their food supply becomes large enough, at $X$, the effect of predation becomes insufficient to control the budworms, and the lower stable equilibrium population disappears. This results in a catastrophic jump in population from $X$ to $W$ to $V$, giving a budworm epidemic. With a budworm epidemic, defoliation is rapid so that the leaf area decreases, but the budworm population does not return to the levels from which it had come. Instead it goes from $V$ to $W$ to $U$, at which point predation takes over from food supply as the limiting factor and the population jumps from $U$ to $Y$.

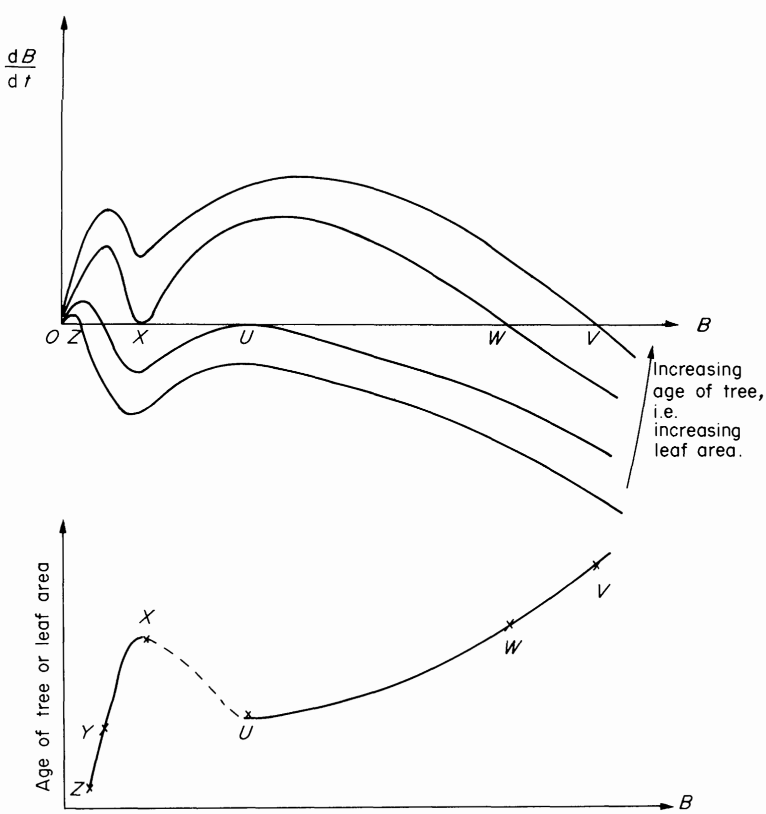


FIG. 4. *Effect of increasing age of trees on budworm population*, B (*derived from Holling*[3]).

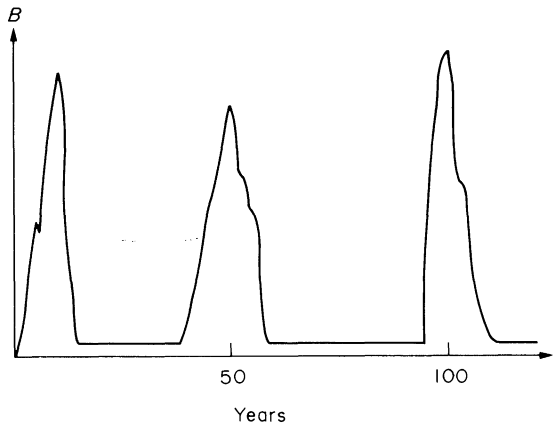


FIG. 5. *Typical historical pattern of number of budworm,* B, *derived from egg counts in tree rings dating back to the early eighteenth century (derived from Holling*[3]).

Historically, the budworm population is illustrated in Figure 5. Although the budworm epidemics die out as quickly as they arise, we have accounted for them by a theory based on low- and high-equilibrium populations. It should be noted that such a theory does not therefore imply that both these equilibria will be exhibited stably by the system for extended periods of time.

We note that the time constant in the budworm population dynamics is much shorter than that for the growth of the trees. Thus this system exhibits the Fold catastrophe, and Figure 4 gives a qualitative description of its behaviour. This can be used to provide qualitative forecasts, e.g. the low level of budworm population will not continue indefinitely in an unmanaged forest: an epidemic will occur at a point determined by tree age and leaf area corresponding to the point $X$. Also results on management strategy can be derived as follows. Spraying with insecticide will not prevent a budworm epidemic occurring: it will simply displace the path $ZYXWV$ to the left. However, spraying can be effective after the epidemic in the region $WU$. If spraying there is sufficient to reduce the budworm level below the unstable equilibrium breakpoint curve (to the left of the line $XU$), then predation will take over and reduce the level to the lower equilibrium on $XY$.

Tree felling is the only way of preventing a budworm outbreak. Trees should be felled before they reach the age corresponding to $X$.

In his more detailed study, Holling[3] separated the variables 'leaf area' and 'tree age' and obtained the Cusp catastrophe. This he used to derive qualitative management strategies, as above, but he was also able to introduce sufficient quantification to specify combinations of the two strategies for different combinations of leaf area and tree age. These strategies were then implemented in coniferous forestry management in New Brunswick. It is noteworthy that this work was based on a model which is extremely efficient in its use of variables, containing only three: budworm population, leaf area and tree age. Even the predator population does not occur explicitly in the model. It has been possible to resist the temptation to build a multidimensional dynamic model of the whole forest ecosystem abounding in non-linear, predator-prey relationships. All the information we require is contained in the variable, $B$. When a catastrophe occurs in $B$, similar catastrophes will occur in many other variables describing different aspects of the forest system. Model simplification, in this sense, is one of the important advantages of catastrophe theory mentioned in the Introduction.

(b) *Fishery management*

As a second example of the use of catastrophe theory in natural resource management, we summarize the work of Peterman[4] on management strategies for a salmon river in Canada.

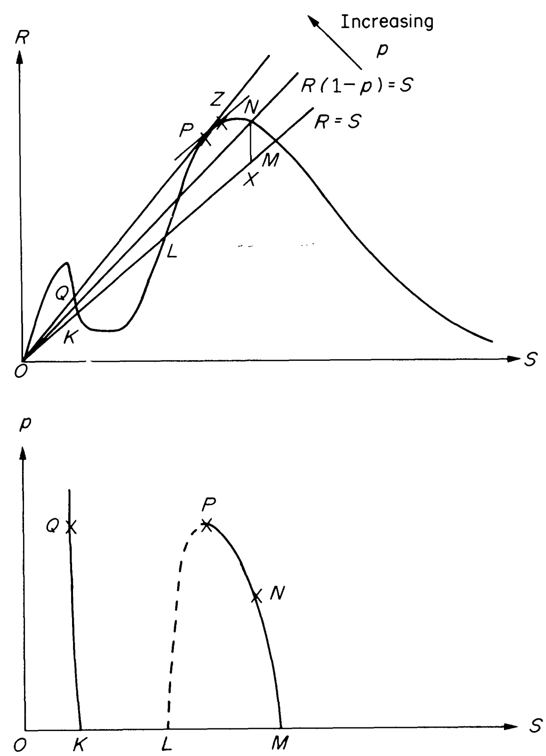


FIG. 6. *Equilibrium numbers of salmon spawners*, S, *and recruits*, R, *for different proportions*, p, *of recruits removed by fishing* (*derived from Peterman*[4]).

Salmon live for two years, dying after spawning eggs. Suppose $S$ salmon spawn eggs. The number of young will be limited by the food supply. Those surviving swim downstream to the ocean, where they are the prey of larger fish. The number, $R$, surviving to return to the spawning grounds after two years, known as recruits, is illustrated qualitatively by Figure 6. The population is in equilibrium if $R = S$. The situation is very similar to the budworm case, with two stable populations $K$, $M$ limited by predation and food supply respectively, and two unstable populations, $O$, $L$.

Now let us suppose that a proportion $p$ of the recruits is removed by fishing. The equilibrium populations are now given by:

$$R(1-p) = S.$$

If the number of salmon originally corresponded to $M$, then as the proportion removed by fishing increases, the system moves from $M$ to $N$ to $P$, at which point the upper equilibrium disappears with a catastrophic jump to $Q$. If now the proportion fished is reduced again, the upper equilibrium salmon population is never regained and the system merely moves from $Q$ to $K$.

Peterman[4] used this model to explain the catastrophic drop in salmon numbers in the Atnarko River between 1961 and 1967 from 2.5 million to 80,000. Also it can be used to forecast that such drops will also occur in future whenever the fishing level approaches point $P$ in Figure 6. Moreover, once the number of fish has reached the lower level, it cannot of its own accord regain the higher level. He also uses it to suggest some qualitative and quantitative management strategies. One such suggestion is similar to those derived by Holling[3] for budworm. As with budworm, there are two possible management strategies for maintaining the upper equilibrium level of salmon, $PNM$: (i) control the level of fishing, (ii) introduce salmon from elsewhere. It is clear from Figure 6 that (ii) is powerless once the proportion fished exceeds the level $P$. Moreover, once the lower equilibrium $QK$ obtains, strategy (i) is powerless without using strategy (ii) as well. In this case, sufficient salmon must be introduced to cross the break point curve, $PL$. Qualitative management advice of this type is very valuable and can be quantified by the collection of the relevant data. However, there is a different type of conclusion that can also be made about the fishery management. The usual strategy for management is based on the concept of maximum sustainable yield (M.S.Y.). The sustainable yield from fishing a proportion $p$ of recruits is $NX$, since $N$ is an equilibrium point and therefore $NX$ fish can be removed each year indefinitely. The maximum sustainable yield thus corresponds to the point $Z$ where the tangent to the curve is parallel to $LM$. It is therefore very dangerous to employ the M.S.Y. strategy for salmon since $Z$ is very near to the point, $P$, where the catastrophe occurs. Peterman calculated that the M.S.Y. value of $p$ was 0.48 and the catastrophic value 0.53. An M.S.Y. strategy should not therefore be employed in this river. In studying the behaviour of equilibrium salmon populations by using catastrophe theory, care should be taken of the point made in the first section about the relative magnitudes of the associated time constants. The time taken for the salmon population to equilibrate may be a few years, as indicated in Figure 7, which gives the historical time series for salmon in the Atnarko River, British Columbia. It is important that the proportion of salmon removed by fishing increased at a rate that was slow compared to this, i.e. changing by only a small amount over a few years. This was in fact the case. If the proportion fished were to be changed suddenly, for instance by a sudden change in licensing policy, some oscillatory behaviour is possible. This serves to accentuate the importance of the last management conclusion mentioned above about the danger of fishing near the M.S.Y. level, since poor weather conditions or residual oscillations in the number of fish can easily cause a catastrophe.

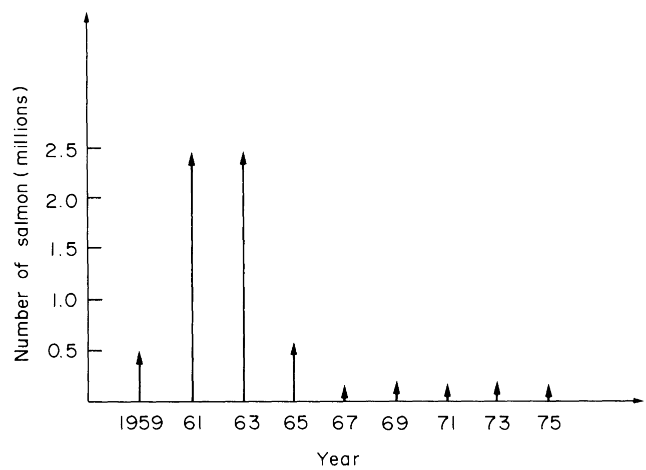


FIG. 7. *Historical time series for salmon in Atnarko River (derived from Peterman*[4]*).*

As with the budworm, modelling a dynamic system by considering upper and lower equilibria does not imply that those equilibria obtain for an extended period of time. This is also clear from Figure 7.

The model has highlighted the two variables of interest, salmon numbers and level of fishing, and does not explicitly contain any others. Such parsimony is a highly desirable advantage of a catastrophe modelling approach and is the main point at which we rely on Thom's theory[1] that the topological form of the catastrophe surface is independent of the dimension of the state variable.

## CONSIDERATIONS REGARDING USE OF CATASTROPHE THEORY IN OTHER AREAS OF MANAGEMENT

In the previous section, we have summarized two case studies of successful applications of catastrophe theory in natural resource management, and in the Introduction we have emphasized certain features of a system that are relevant to the possibility of using catastrophe theory on it. Two key features are (i) the presence of upper and lower stable equilibria and (ii) fast and slow time constants for the response of the system to different variables.

These two features are present in many other practical areas of management. In order to exemplify the potential usefulness of catastrophe theory, we now review two management systems which exhibit these features. They are described as case studies by Coyle.[5,6]

(a) *Chartering oil tankers*

This case study concerns the scheduling of charters for oil tankers to transport crude oil from the Middle East to meet a seasonal demand in North West Europe.

There are basically two ways in which oil tankers can be chartered: (i) spot chartering, where a tanker is chartered for a single trip and presents itself for service within one day, (ii) time chartering, where a tanker is chartered for three to five years and presents itself for service within one month. The prices obtaining in these chartering markets in the Middle East are notoriously volatile, and a historical time series is given in Figure 8 for a period not affected by local political problems.

The two different types of chartering affect the number of ships demanded with very different time constants, and the peaks in Figure 8 are similar to those in Figures 5 and 7 and could be evidence of catastrophic jumps between upper and lower equilibrium levels.

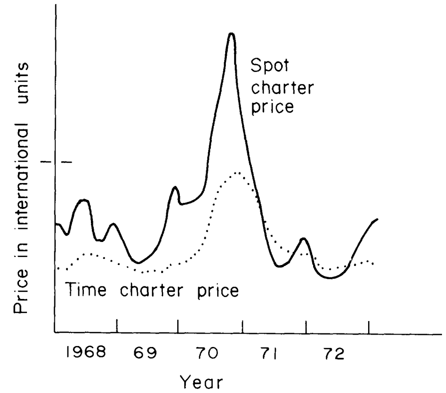


FIG. 8. *Historical time series for prices of chartering oil tankers in the Middle East (derived from Coyle*[5]*).*

(b) *Scheduling expenditure on capital projects*

This case study concerns corporate planning of an international company concerned with allocating central funds to various capital projects.

The allocation of funds to capital projects evidently affects those funds with a long time constant of the order of several years. Also funds can be supplemented by borrowing, which affects them with a much shorter time constant. Capital expenditure, particularly on new projects, exhibits a cyclical pattern whose relative amplitude is two to three times as large as that of other variables, e.g. G.N.P., in the business cycle. This is, in itself, evidence of some instability in the capital market, but the purpose of Coyle's model[6] is to show that with a gradual increase in raw material prices, capital expenditure becomes extremely unstable, oscillating orders of magnitude more than G.N.P.

This again can therefore be regarded as a catastrophe in a system controlled by the two different time constants mentioned above.

The advantages of the use of catastrophe theory in these and other areas would be: (i) a reduction in the number of variables in the model, (ii) the possibility of quickly obtaining qualitative results on management strategy without requiring much quantitative data, (iii) the possibility of quantifying these qualitative results if sufficient quantitative data exist.

## CONCLUSIONS

Two case studies have been presented giving practical examples of the successful implementation of catastrophe theory models in management decision making. Examples of other management application areas where catastrophe theory is potentially useful have also been described. The features of a system that render catastrophe theory applicable to it are: (i) the existence of more than one stable equilibrium state, even though such equilibria need not persist for extended periods of time: (ii) the existence of a variable governed by at least two subsystems with significantly differing time constants. The advantages of using catastrophe theory to model such a system are (i) the isolation of certain key variables into which other variables have been subsumed, (ii) the ability to recommend qualitative management decisions based on a qualitative description of the system, (iii) the possibility of indicating what quantitative data need to be made available in order to quantify the qualitative conclusions.

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